Kinematics

Control and Robotics in Medicine

Departamento de
Tecnología Fotónica
y Bioingeniería
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DOFs - Definition

- **DOF**: Minimum number of independent kinematic variables that allows a mechanism to move.
- **DOF**: Number of independent motions that are allowed to the end effector of a mechanism.

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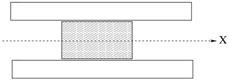

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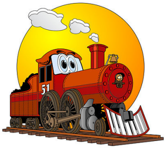

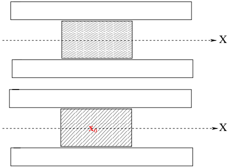

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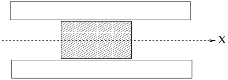

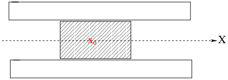

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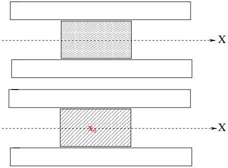

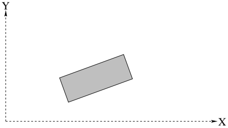

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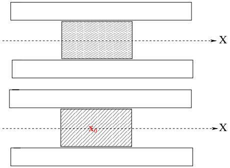

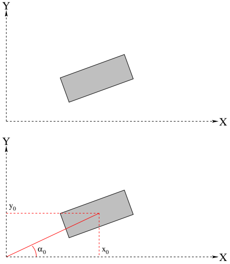

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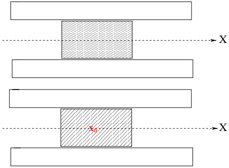

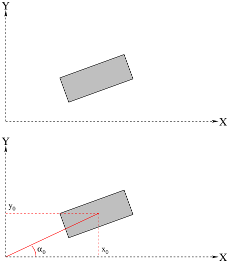

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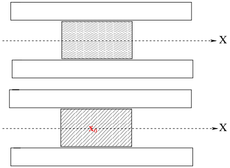

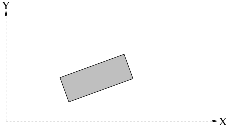

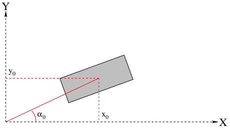

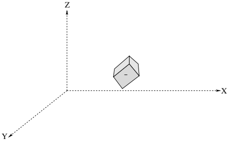

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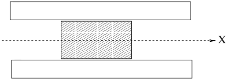
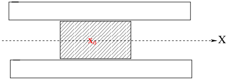
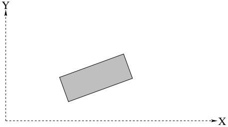
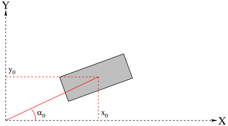
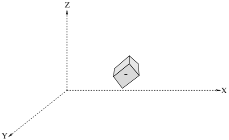
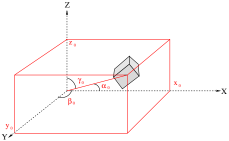




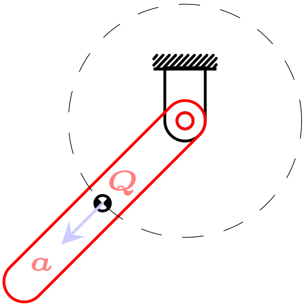








DOFs - Manual



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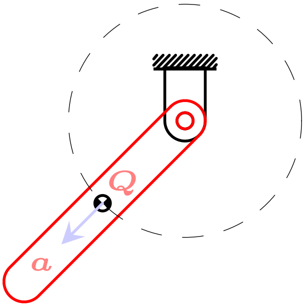

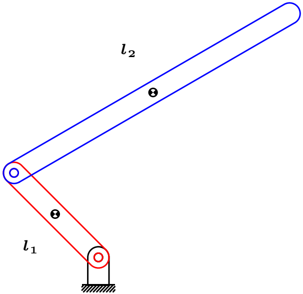

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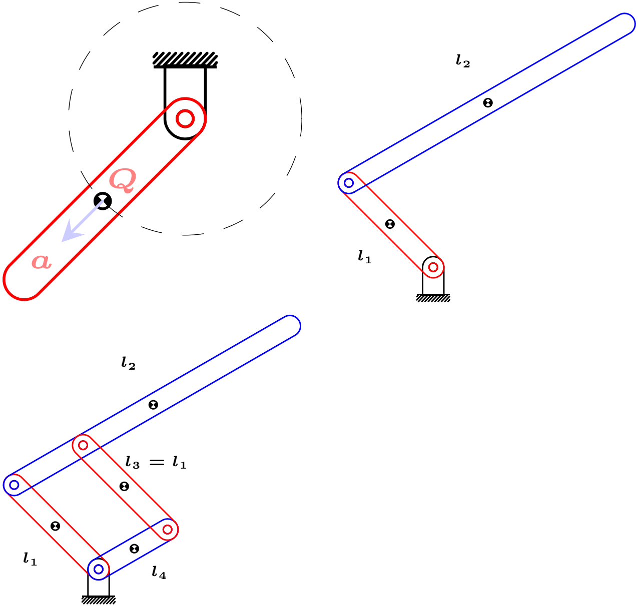

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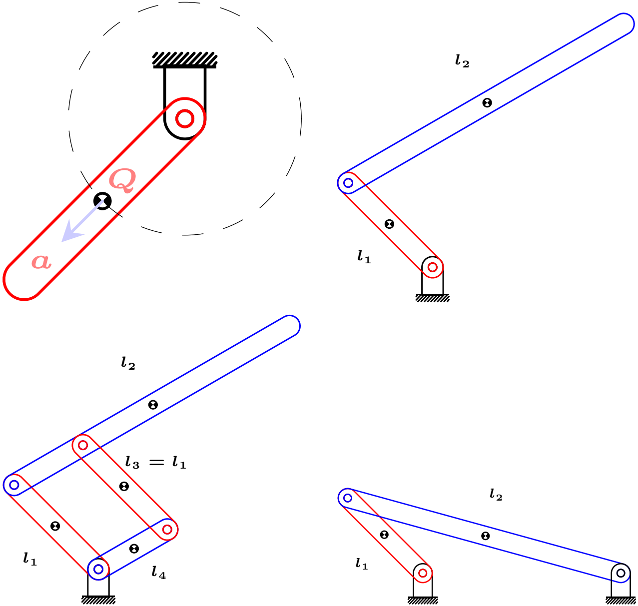

DOFs - Formules

- Implicit Function Theorem in $\mathbb{R}^3$

$$n = 6N - m \tag{1}$$

  - N is the number of rigid bodies
  - m is the number of constraints

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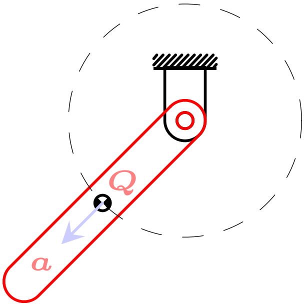
  - N is the number of rigid bodies
  - m is the number of constraints

- Grübler-Kutzbach mobility criteria

$$n = \mathcal{D}(\mathcal{N} - 1 - j) + \sum_{i=1}^{j} f_i \tag{2}$$

  - $\mathcal{D}$ is the space dimension ($\mathcal{D} = 6$ in a three-dimensional space and $\mathcal{D} = 3$ in the plane),
  - $\mathcal{N}$ is the number of rigid bodies (anchor to the ground included)
  - j is the number of joints
  - f_i is the number of DOFs of the $i - th$ joint.

DOFs - Implicit Function Theorem



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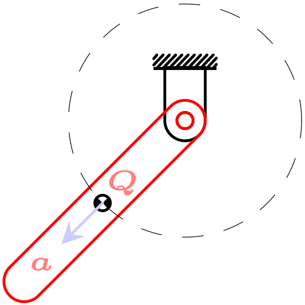

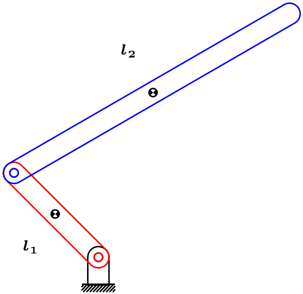

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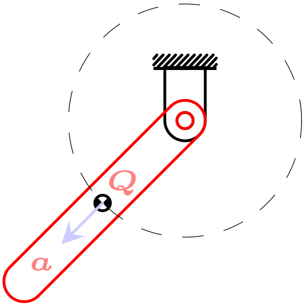

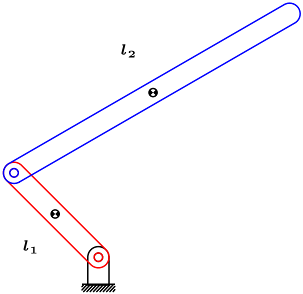

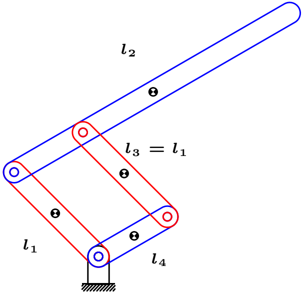

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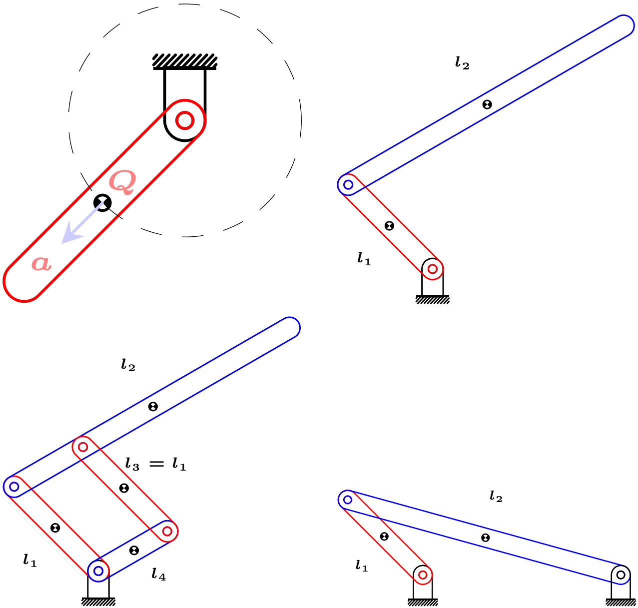
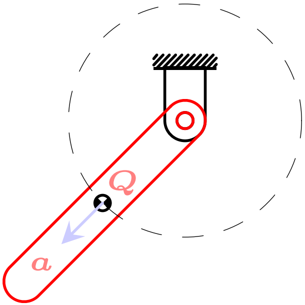


DOFs - Grübler-Kutzbach mobility criteria



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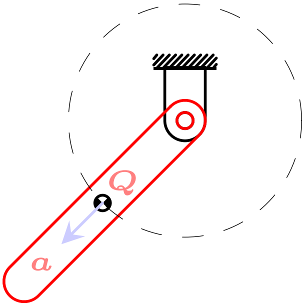

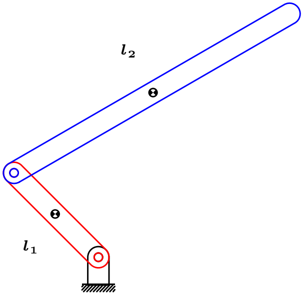

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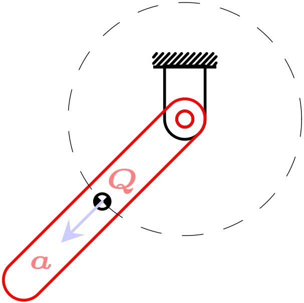

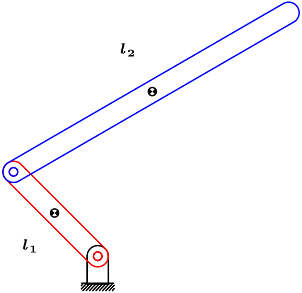

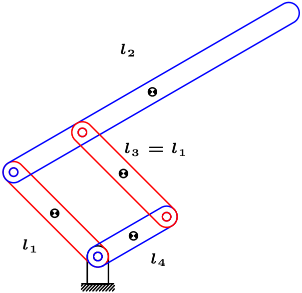

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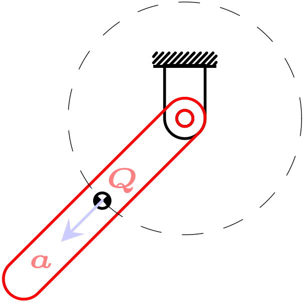
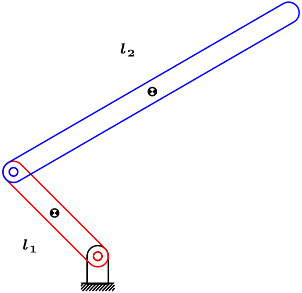
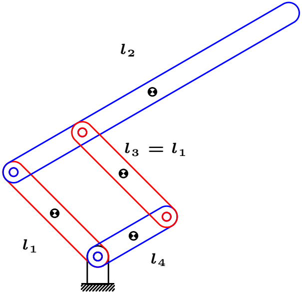
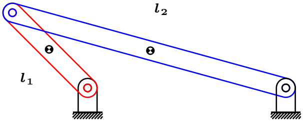
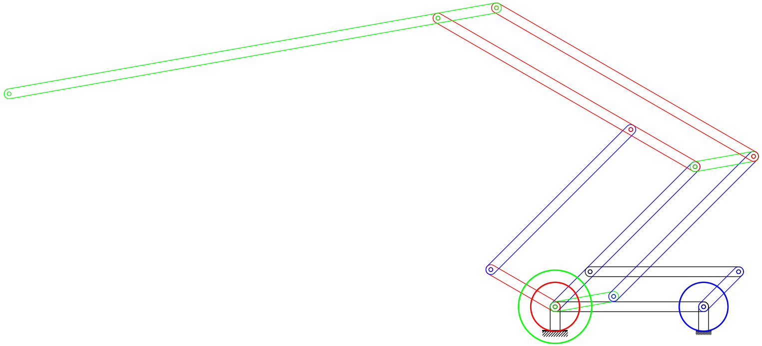








DOFs - HomeWork



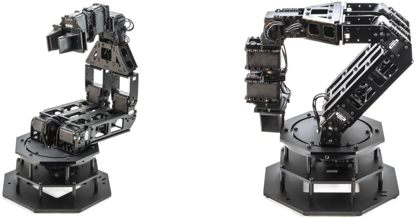
1. Degrees of Freedom
2. Forward kinematics
3. Inverse kinematics
4. Deliverable D1

Definition

- **The use of the kinematic equations of a robot to compute the position ($[x\ y\ z]^T$) and orientation (R) of the end-effector (hand's end) from specified values for the joint parameters or generalized coordinates (q)**

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = f(q)$$

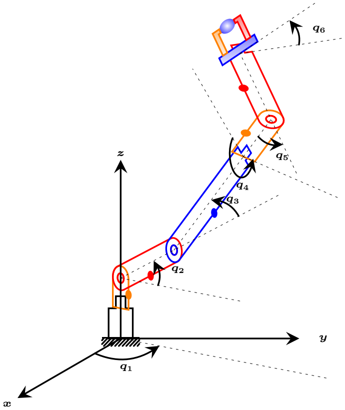
$$R = g(q)$$



Parameters

- **Generalized coordinates**: The parameters that describe the configuration of the system relative to some reference configuration.

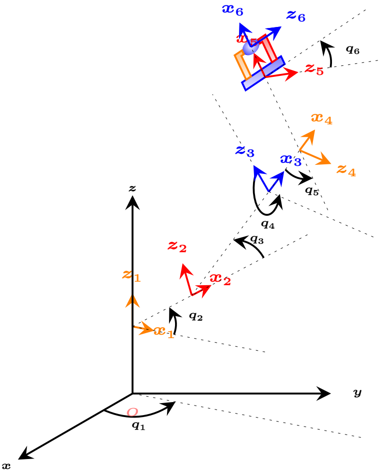
$$q = \{q_1, q_2, q_3, q_4, q_5, q_6\} \tag{3}$$



Parameters

- **Rotation Axes**: The vector of rotations of every kinematic chain.

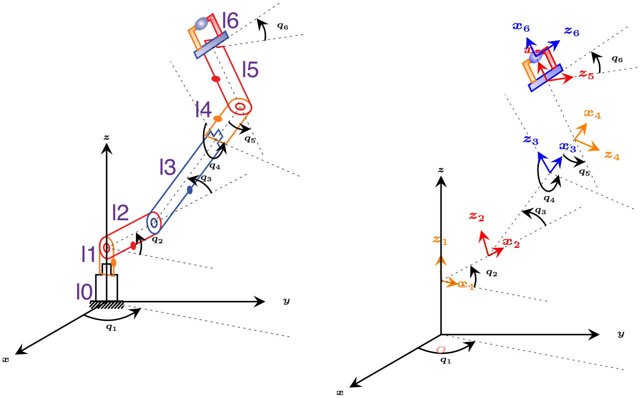
$$\mathcal{U} = \{z_0, -y_1, -y_2, x_3, y_4, x_5\} \quad (4)$$



Parameters

- **Local references**: Joint related to the reference system prior to rotation.
- $\widetilde{o}_i$ represents the origin of every local reference system.

$$\widetilde{\mathcal{O}} = \{(l_0 + l_1)z_0, l_2x_1, l_3x_2, l_4x_3, l_5x_4, l_6x_5\} \tag{5}$$



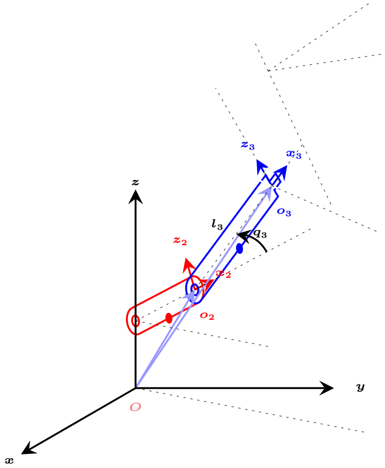
Rotations and translations

- Every point P in the three-dimensional Cartesian space can be represented by coordinates related to any reference system.

$$p_0 = R_0^1 p_1 + t_0^1 \tag{6}$$

- p_0 represents the coordinates of point P related to $\mathcal{S}_0$
- p_1 represents the coordinates of the same point P related to $\mathcal{S}_1$
- t_0^1 represents the translation vector defined in $\mathcal{S}_0$.
- R_0^1 represents the rotation matrix of $\mathcal{S}_1$ related to $\mathcal{S}_0$.

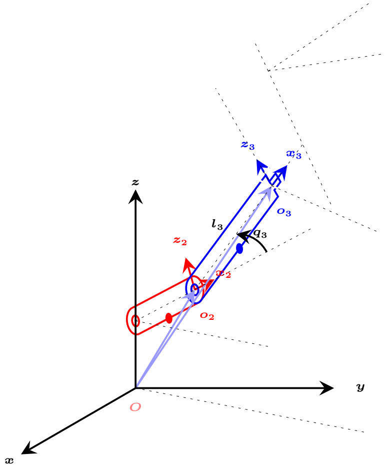
Rotations and translations



$$d = \begin{bmatrix} x & y & z \end{bmatrix}^T$$

$$d_{i-1}^{i} = \overrightarrow{o_{i-1}o_i}$$

Rotations and translations



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$$\text{e.g.: } d_2^3 = \overrightarrow{o_2 o_3}$$

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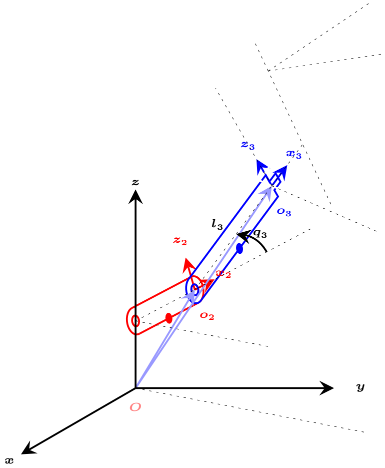

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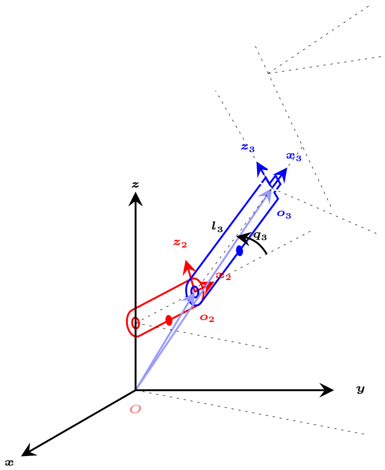

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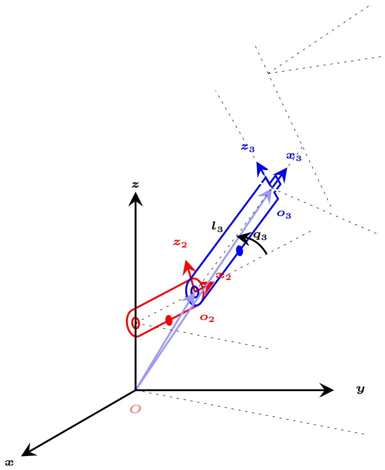

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$$R_{x,\theta} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix} \quad R_{y,\theta} = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \quad R_{z,\theta} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Forward kinematics

- Fundamental equation of the forward kinematics of position

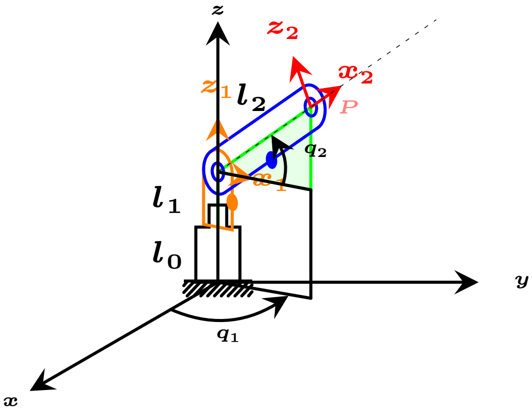
$$d_0^i = d_0^{i-1} + R_0^{i-1} d_{i-1}^i \tag{7}$$

- Because the composition of rotations of local reference systems satisfies the chain rule, then:

- Fundamental equation of the forward kinematics of orientation

$$R_0^i = R_0^{i-1} R_{i-1}^i \tag{8}$$

Forward kinematics - Example



$$d_0^i = d_0^{i-1} + R_0^{i-1} d_{i-1}^i \quad (9)$$

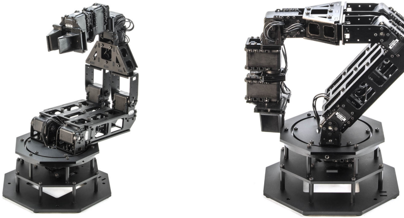
$$R_0^i = R_0^{i-1} R_{i-1}^i \quad (10)$$

$$d_{i-1}^i = R_{i-1}^i \tilde{o}_i \quad (11)$$

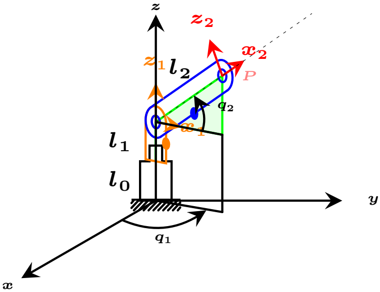
Definition

- **The use of the kinematic equations of a robot to compute the generalized coordinates (*q*) from the position ($[x\ y\ z]^T$) and orientation (*R*) of the end-effector (hand's end)**

$$q = f\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}, R\right)$$



Inverse kinematics - Example



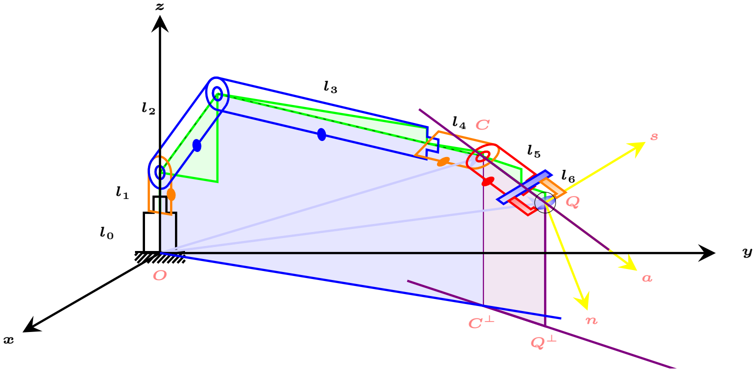
$$d_0^2 = (l_0 + l_1)\begin{bmatrix}0\\0\\1\end{bmatrix} + l_2\begin{bmatrix}cq_1 cq_2\\sq_1 cq_2\\sq_2\end{bmatrix} = \begin{bmatrix}x\\y\\z\end{bmatrix} \quad (12)$$

$$x = l_2 cq_1 cq_2 \quad (13)$$

$$y = l_2 sq_1 cq_2 \quad (14)$$

$$z = (l_0 + l_1) + l_2 sq_2 \quad (15)$$

Kinematic decoupling

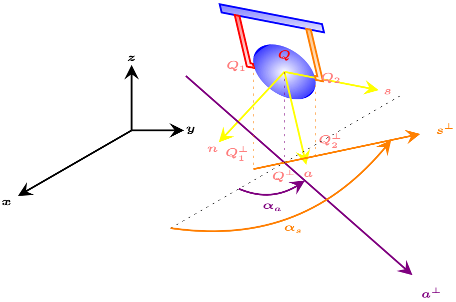


Orientation vector

- **Hand's end orientation**: approach (a), normal (n) and sliding (s):

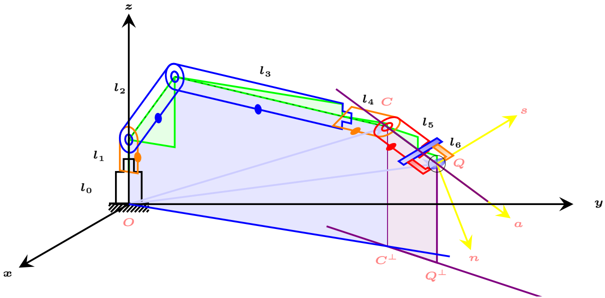
$$a = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} \quad n = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} \quad s = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$$

$$R_{\mathcal{S}_O x} = \begin{bmatrix} a_x & -n_x & -s_x \\ a_y & -n_y & -s_y \\ a_z & -n_z & -s_z \end{bmatrix}$$

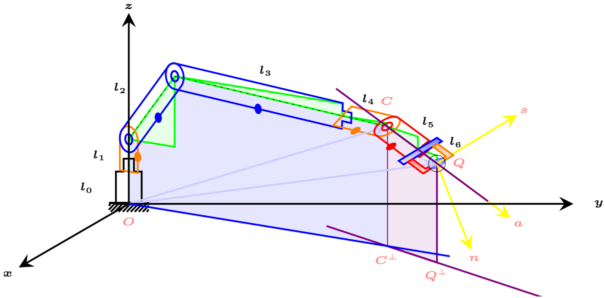


Inverse kinematics - Procedure

1. Obtain the wrist center C: $\overrightarrow{OC} = \overrightarrow{OQ} - d_H\ \vec{a}$
2. Solve the inverse kinematics problem of position C to obtain $\{q_1, q_2, q_3\}$.
3. Calculate the rotation matrix $R_0^3 = R_{\mathcal{S}_A}$.
4. Calculate $R_{\mathcal{S}_H} = R_{\mathcal{S}_A}^T R_{\mathcal{S}_0}$
5. Solve the inverse kinematics problem of orientation R_3^6 to obtain $\{q_4, q_5, q_6\}$.



Inverse kinematics of position

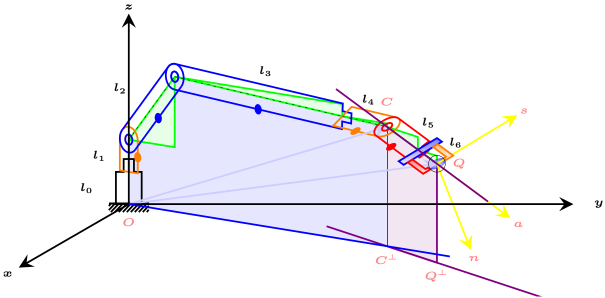


$$Q = \begin{bmatrix} Q_x & Q_y & Q_z \end{bmatrix}^T \quad C = \begin{bmatrix} C_x & C_y & C_z \end{bmatrix}^T \quad \overrightarrow{OC} = \overrightarrow{OQ} - (l_5 + l_6)\ \vec{a}$$

$$e.g.:\ a = \begin{bmatrix} a_x & -n_x & -s_x \\ a_y & -n_y & -s_y \\ a_z & -n_z & -s_z \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\overrightarrow{OC} = \overrightarrow{OQ} - (l_5 + l_6) R_{\mathcal{S}_O} \vec{i}$$

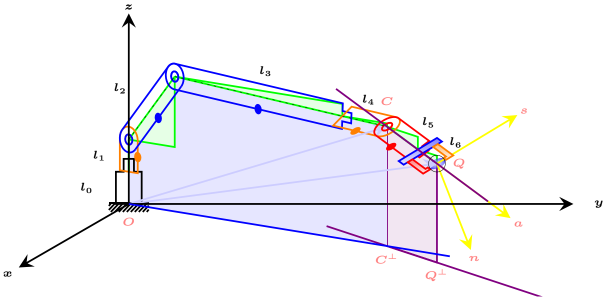
Inverse kinematics of position



$$\begin{bmatrix} C_x \\ C_y \\ C_z \end{bmatrix} = \begin{bmatrix} f(q_1, q_2, q_3) \\ g(q_1, q_2, q_3) \\ h(q_1, q_2, q_3) \end{bmatrix}$$

$$\begin{cases} C_x = f(q_1, q_2, q_3) \\ C_y = g(q_1, q_2, q_3) \\ C_z = h(q_1, q_2, q_3) \end{cases}$$

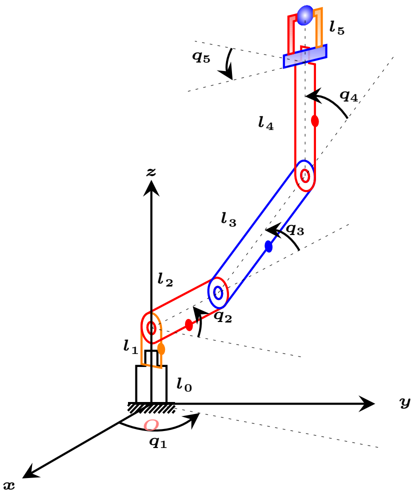
Inverse kinematics of orientation

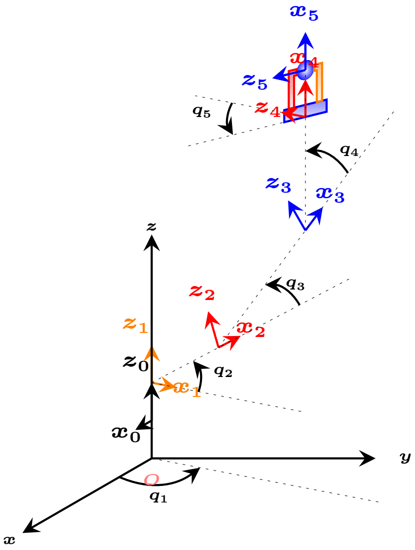


- $q_1, q_2, q_3 \rightarrow R_0^3 = R_{\mathcal{S}_A}$
- $R_{\mathcal{S}_O} = R_0^3 R_3^6$
- $R_{\mathcal{S}_O} = R_{\mathcal{S}_A} R_{\mathcal{S}_H} \rightarrow R_{\mathcal{S}_H} = R_{\mathcal{S}_A}^T R_{\mathcal{S}_O}$

$$R_3^6 = F(q_4, q_5, q_6) = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{bmatrix}$$

Deliverable D1





Deliverable D1

1. **Formulate the forward kinematics problem in position and orientation (25 %).**
2. **Inverse kinematics problem (75 %).**
   2.1 **Formulate the inverse kinematics problem by using the kinematic decoupling technique**
   2.2 **Solve the inverse kinematics problem when** $Q(t_g) = (210, 30, 150)$, $a(t_g) = \begin{bmatrix} 0.9899 & 0.1414 & 0 \end{bmatrix}^T$ **and** $s(t_g) = \begin{bmatrix} -0.1414 & 0.9899 & 0 \end{bmatrix}^T$.
   2.3 **Solve the inverse kinematics problem when** $Q(t_r) = (20, 170, 150)$, $a(t_r) = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$ **and** $s(t_r) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.

THANKS FOR LISTENING!!