Trajectory Planning

Control and Robotics in Medicine

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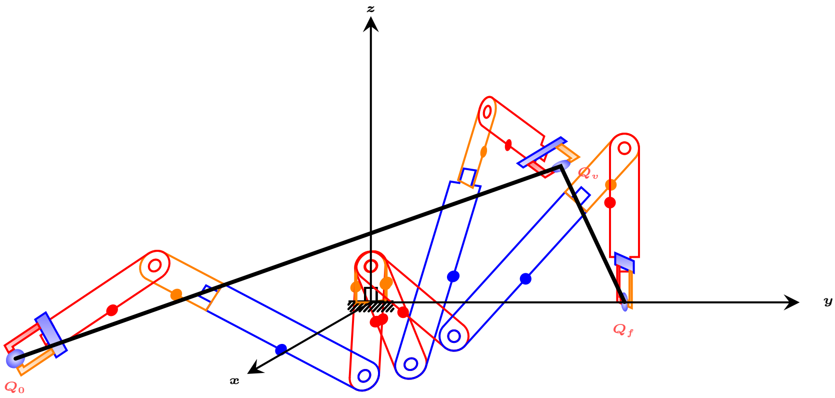
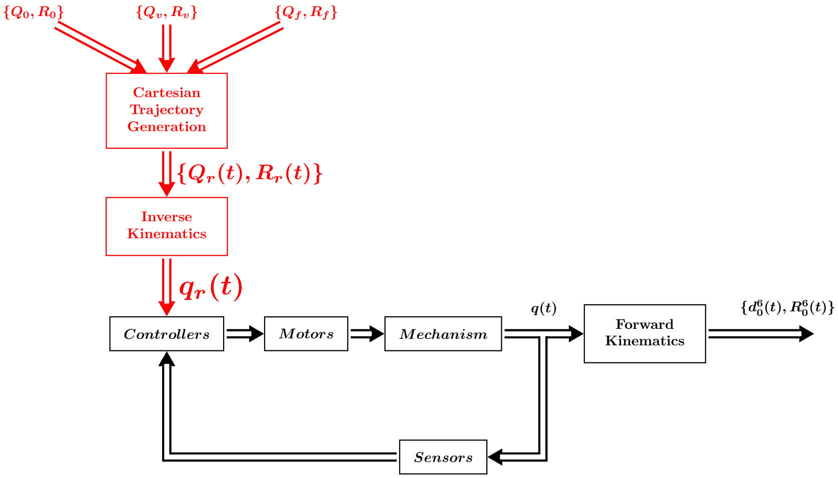
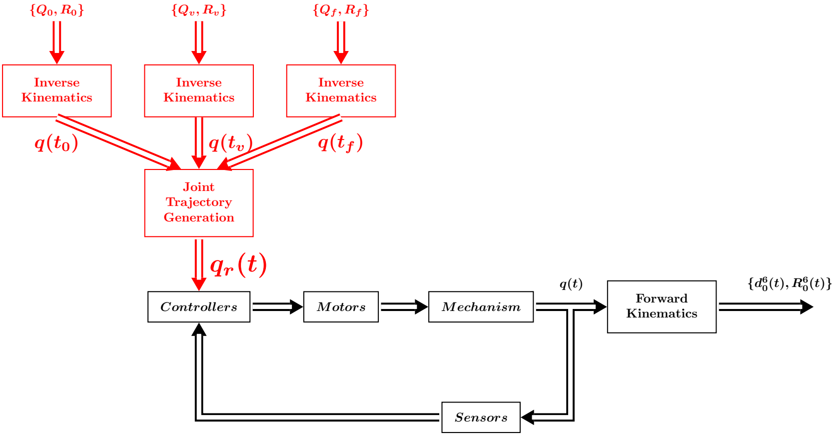
Introduction - Trajectory



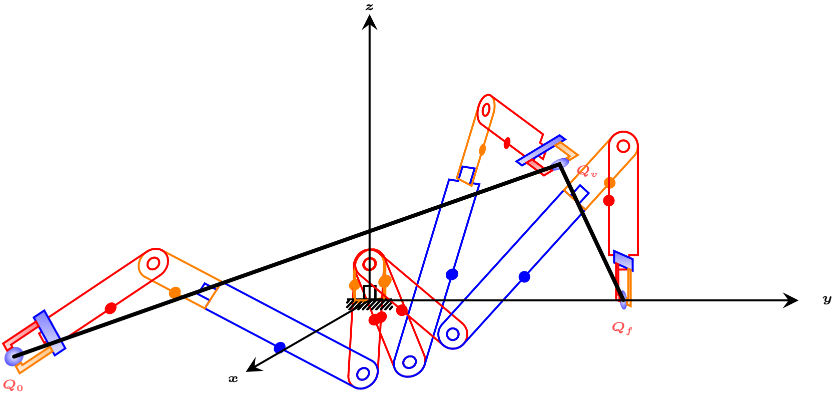
Introduction - Cartesian Space



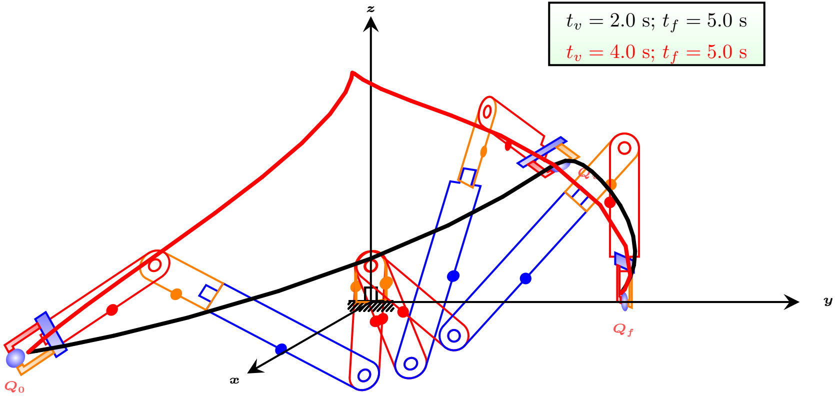
Introduction - Joint Space



Introduction - Example



Introduction - Example



Definitions

- Generalized coordinates

$$q_i(t) = a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3$$

- Restrictions

$$\mathcal{C} = \{t_0, t_f, q_i(t_0), q_i(t_f), \dot{q}_i(t_0), \dot{q}_i(t_f)\}$$

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- Solve the system

$$q_i(t_0) = a_{i0} + a_{i1}t_0 + a_{i2}t_0^2 + a_{i3}t_0^3 \quad (1)$$

$$q_i(t_f) = a_{i0} + a_{i1}t_f + a_{i2}t_f^2 + a_{i3}t_f^3 \quad (2)$$

$$\dot{q}_i(t_0) = a_{i1} + 2a_{i2}t_0 + 3a_{i3}t_0^2 \quad (3)$$

$$\dot{q}_i(t_f) = a_{i1} + 2a_{i2}t_f + 3a_{i3}t_f^2 \quad (4)$$

Example

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$$q_i(t) = a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3$$

- Restrictions

$$\mathcal{C} = \{t_0, t_f, q_i(t_0), q_i(t_f), \dot{q}_i(t_0), \dot{q}_i(t_f)\} \rightarrow \{0, t_f, q_i(0), q_i(t_f), 0, 0\}$$

Example

- Generalized coordinates

$$q_i(t) = a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3$$

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- Solve the system

$$q_i(t_0) = a_{i0} + a_{i1}t_0 + a_{i2}t_0^2 + a_{i3}t_0^3 \qquad \rightarrow a_{i0} \qquad (5)$$

$$q_i(t_f) = a_{i0} + a_{i1}t_f + a_{i2}t_f^2 + a_{i3}t_f^3 \qquad (6)$$

$$\dot{q}_i(t_0) = a_{i1} + 2a_{i2}t_0 + 3a_{i3}t_0^2 \qquad \rightarrow 0 \qquad (7)$$

$$\dot{q}_i(t_f) = a_{i1} + 2a_{i2}t_f + 3a_{i3}t_f^2 \qquad \rightarrow 0 \qquad (8)$$

Example

- Continuous

$$q_i(t) = q_i(0) + 3\alpha_i t_f t^2 - 2\alpha_i t^3 \quad (9)$$

$$\alpha_i = \frac{q_i(t_f) - q_i(0)}{t_f^3} \quad (10)$$

- Discrete: $t_f = NT$ and $t = kT$

$$q_i(k) = q_i(0) + \beta_i(3N - 2k)k^2 \quad (11)$$

$$\beta_i = \frac{q_i(N) - q_i(0)}{N^3} = T^3\alpha_i \quad (12)$$

Definitions

- Generalized coordinates

$$q_i(t) = \begin{cases} a_{i0} + a_{i1}t + a_{i2}t^2 + a_{i3}t^3 \quad , \quad t \in [t_0, t_v] \\ b_{i0} + b_{i1}t + b_{i2}t^2 + b_{i3}t^3 \quad , \quad t \in [t_v, t_f] \end{cases}$$

- Restrictions

$$\mathcal{C} = \{t_0, t_v, t_f, q_i(t_0), q_i(t_v), q_i(t_f), \dot{q}_i(t_0), \dot{q}_i(t_f)\}$$

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- Solve the system

$$q_i(t_0) = a_{i0} + a_{i1}t_0 + a_{i2}t_0^2 + a_{i3}t_0^3 \qquad \dot{q}_i(t_0) = a_{i1} + 2a_{i2}t_0 + 3a_{i3}t_0^2$$

$$q_i(t_f) = b_{i0} + b_{i1}t_f + b_{i2}t_f^2 + b_{i3}t_f^3 \qquad \dot{q}_i(t_f) = b_{i1} + 2b_{i2}t_f + 3b_{i3}t_f^2$$

$$q_i(t_v) = a_{i0} + a_{i1}t_v + a_{i2}t_v^2 + a_{i3}t_v^3 \qquad \dot{q}_i(t_v) = a_{i1} + 2a_{i2}t_v + 3a_{i3}t_v^2$$

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Definitions

- Restrictions

$$\mathcal{C} = \{t_0, t_v, t_f, q_i(t_0), q_i(t_v), q_i(t_f), 0, 0\}$$

- Generalized coordinates

$$q_i(t) = \begin{cases} a_{i0} + a_{i1}(t - t_0) + a_{i2}(t - t_0)^2 + a_{i3}(t - t_0)^3 & , \quad t \in [t_0, t_v] \\ b_{i0} + b_{i1}(t_f - t) + b_{i2}(t_f - t)^2 + b_{i3}(t_f - t)^3 & , \quad t \in [t_v, t_f] \end{cases}$$

Definitions

- Restrictions

$$\mathcal{C} = \{t_0, t_v, t_f, q_i(t_0), q_i(t_v), q_i(t_f), 0, 0\}$$

- Generalized coordinates

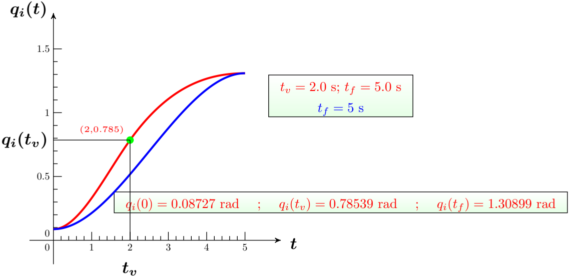
$$q_i(t) = \begin{cases} a_{i0} + a_{i1}(t - t_0) + a_{i2}(t - t_0)^2 + a_{i3}(t - t_0)^3 & , \quad t \in [t_0, t_v] \\ b_{i0} + b_{i1}(t_f - t) + b_{i2}(t_f - t)^2 + b_{i3}(t_f - t)^3 & , \quad t \in [t_v, t_f] \end{cases}$$

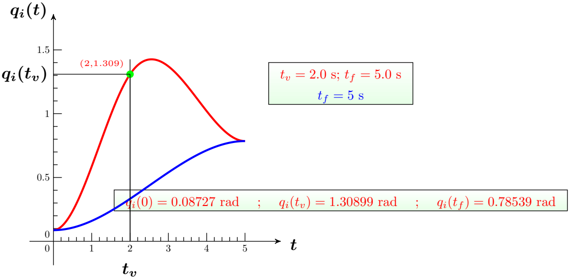
- Solve the system

$$a_{i0} = q_i(t_0)$$
$$b_{i0} = q_i(t_f)$$
$$a_{i1} = 0$$
$$b_{i1} = 0$$

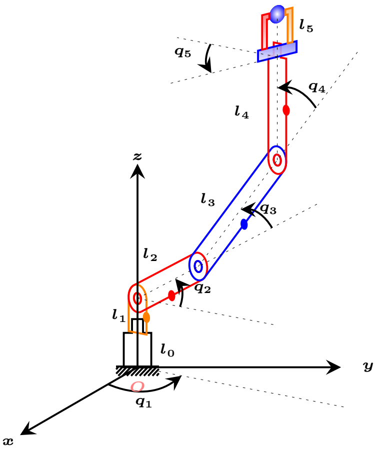
$$q_i(t) = \begin{cases} q_i(t_0) + a_{i2}(t - t_0)^2 + a_{i3}(t - t_0)^3 & , \quad t \in [t_0, t_v] \\ q_i(t_f) + b_{i2}(t_f - t)^2 + b_{i3}(t_f - t)^3 & , \quad t \in [t_v, t_f] \end{cases}$$

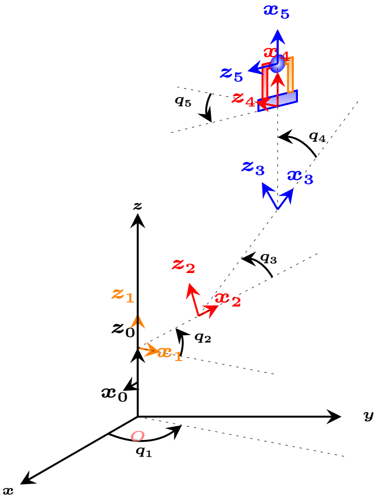
Cubic GC graphs





Deliverable D2



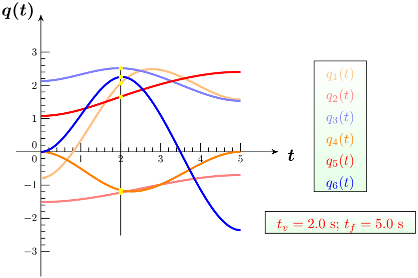


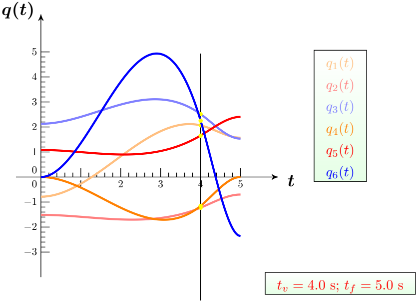
Deliverable D2

1. **Trajectory planning (45%)**.
   1.1 **Cubic trajectory planning of joints with zero velocity in the initial and final points** (40%): $q(t_0) = \{0, 2.89, -2.89, 0, 0\}$; $Q(t_g) = (210, 30, 150)$, $a(t_g) = \begin{bmatrix} 0.9899 & 0.1414 & 0 \end{bmatrix}^T$ and $s(t_g) = \begin{bmatrix} -0.1414 & 0.9899 & 0 \end{bmatrix}^T$
   1.2 **Cubic trajectory planning of joints with zero velocity in the initial and final point passing by an intermediate point** (60%): $Q(t_g) = (210, 30, 150)$, $a(t_g) = \begin{bmatrix} 0.9899 & 0.1414 & 0 \end{bmatrix}^T$ and $s(t_g) = \begin{bmatrix} -0.1414 & 0.9899 & 0 \end{bmatrix}^T$; $Q(t_r) = (20, 170, 150)$, $a(t_r) = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}^T$ and $s(t_r) = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}^T$.
   $Q(t_v) = (x_v, y_v, z_v)$, $a(t_v)$, $s(t_v)$ and $n(t_v)$?? $\rightarrow Q_O = (130, 100, 0)$

2. **Implementation on the real robot (50%)**.
   2.1 **Implementation on the real robot of the trajectory defined in Part 1** with the grasping of an object (40%).
   2.2 **Implementation on the real robot of the trajectory defined in Part 2** with the release of an object (60%).
3. **Conclusions (5%)**.
4. **References**

Deliverable D2





Laboratory

http://wiki.robolabo.etsit.upm.es/index.php/PhantomX_Reactor_Robot

Computer installation

- Requisites
  - Operating System independency
  - 1 USB

Arduino IDE

- Download and install last IDE version
  - https://www.arduino.cc/en/Main/Software

Download the Arduino IDE

ARDUINO 1.8.1

The open-source Arduino Software (IDE) makes it easy to write code and upload it to the board. It runs on Windows, Mac OS X, and Linux. The environment is written in Java and based on Processing and other open-source software.
This software can be used with any Arduino board.
Refer to the Getting Started page for Installation instructions.

Windows Installer
Windows ZIP file for non admin install

Windows app Get

Mac OS X 10.7 Lion or newer

Linux 32 bits
Linux 64 bits
Linux ARM

Release Notes
Source Code
Checksums (sha512)

ARDUINO SOFTWARE
HOURLY BUILDS

LAST UPDATE
25 January 2017 16:37:31 GMT

Download a preview of the incoming release with the most updated features and bugfixes.

Windows
Mac OS X (Mac OSX Lion or later)
Linux 32 bit , Linux 64 bit , Linux ARM

ARDUINO 1.0.6 / 1.5.x / 1.6.x
PREVIOUS RELEASES

Download the previous version of the current release, the classic Arduino 1.0.x, or the Arduino 1.5.x Beta version.

All the Arduino 00xx versions are also available for download. The Arduino IDE can be used on Windows, Linux (both 32 and 64 bits), and Mac OS X.

Robot libraries

- Download and install arbotix libraries
  - Install Hello World example
  - Check compilation and study the code
  - Install P1 (D2) template

Code libraries

- **void ROBOT_GripperOpen(void)**: opens the gripper
- **void ROBOT_GripperClose(void)**: closes the gripper
- **void ROBOT_SetSingleTrajectory (double *f_pos, uint16_t un_time, uint8_t un_trajectory_type))**
  - f_pos = $q(t_f)$; un_time = $t_f - t_0$
  - un_trajectory_type = LINEAR or CUBIC1
- **void ROBOT_SetDoubleTrajectory (double *un_pos1, double *un_pos2, uint16_t un_time1, uint16_t un_time2, uint8_t un_trajectory_type)**
  - f_pos1 = $q(t_v)$; f_pos2 = $q(t_f)$;
  - un_time1 = $t_v - t_0$; un_time2 = $t_f - t_v$
  - un_trajectory_type = CUBIC2

THANKS FOR LISTENING!!